Section 9.1 Multiplication Properties of Exponents

1. Property One for Exponents: If r and s are any two whole numbers and a is an integer, then it is true that:

$$a^r \bullet a^s = a^{r+s}$$

Example 1: Simplify each of the following.

a. $x^2x^4 = x^{2+4}$
$= x^6$

b. $x^3x^5 = x^{3+5}$
$= x^8$

c. $2x^2 \bullet 3x^4 = 2 \cdot 3 \cdot x^2 \cdot x^4$
$= 6 \cdot x^{2+4}$
$= 6x^6$

2. Property Two for Exponents: If r and s are any two whole numbers and a is an integer, then it is true that:

$$\left(a^r\right)^s = a^{rs}$$

Example 2: Simplify each of the following.

a. $\left(x^3\right)^4 = x^{3 \cdot 4}$
$= x^{12}$

b. $2\left(x^3\right)^5 = 2 \cdot x^{3 \cdot 5}$
$= 2x^{15}$

3. Property Three for Exponents: If r is a whole number and a and b are integers, then it is true that:

$$(ab)^r = a^r b^r$$

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SDWK

$2^3 = 2 \cdot 2 \cdot 2$
$= 8$

$4^2 = 4 \cdot 4$
$= 16$

Example 3: Simplify each of the following.

a. $(2x)^3 = 2^3 \cdot x^3$
$= 8x^3$

b. $(4x^5)^2 = 4^2 \cdot (x^5)^2$
$= 16 \cdot x^{5 \cdot 2}$
$= 16x^{10}$

4. Simplifying Using More Than One Property: Use the order of operations agreement and the three multiplication properties of exponents to simplify.

Example 4: In each of the following identify the property used in each step.

a. $(2x^2y^3)(3x^5y^4) = (2 \bullet 3)(x^2x^5)(y^3y^4)$
$= 6x^7y^7$

b. $(3x^3y^2)^4 = 3^4 \bullet (x^3)^4 \bullet (y^2)^4$
$= 81x^{12}y^8$

SDWK

$4^2 = 4 \cdot 4$
$= 16$

$3^2 = 3 \cdot 3$
$= 9$
$2^3 = 2 \cdot 2 \cdot 2$
$= 8$

Example 5: Simplify each of the following.

a. $(4x^3y^5)^2 = (4)^2 \cdot (x^3)^2 \cdot (y^5)^2$
$= 16 \cdot x^{3 \cdot 2} \cdot y^{5 \cdot 2}$
$= 16x^6y^{10}$

b. $(3x^2y)^2(2x^3y^2)^3$
$= (3)^2 \cdot (x^2)^2 \cdot y^2 \cdot (2)^3 \cdot (x^3)^3 \cdot (y^2)^3$
$= 9 \cdot x^{2 \cdot 2} \cdot y^2 \cdot 8 \cdot x^{3 \cdot 3} \cdot y^{2 \cdot 3}$
$= 9 \cdot 8\, x^4 \cdot y^2 \cdot x^9 y^6$
$= 72 \cdot x^{4+9} \cdot y^{2+6}$
$= 72x^{13}y^8$

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Practice Problems. Use the Multiplication Properties of Exponents to simplify each of the following:

a. $4x^3 \bullet 5x^5 = 4 \cdot 5 \cdot x^3 \cdot x^5$
$= 20\ x^{3+5}$
$= 20x^8$

b. $\left(a^4\right)^5 = a^{4 \cdot 5}$
$= a^{20}$

SDWK
$5^3 = 5 \cdot 5 \cdot 5$
$= 25 \cdot 5$
$= 125$

c. $(5x)^3 = 5^3 \cdot x^3$
$= 125\,x^3$

SDWK
$2^2 = 2 \cdot 2$
$= 4$

d. $\left(2x^3\right)^2\left(4x^5\right)$
$= (2)^2 \cdot (x^3)^2 \cdot 4 \cdot x^5$
$= 4 \cdot x^{3 \cdot 2} \cdot 4 \cdot x^5$
$= 4 \cdot 4 \cdot x^6 \cdot x^5$
$= 16\ x^{6+5}$
$= 16\ x^{11}$

e. $\left(2xy^2\right)\left(5x^3y^4\right)$
$= 2 \cdot 5 \cdot x^1 \cdot x^3 \cdot y^2 \cdot y^4$
$= 10 \cdot x^{1+3} \cdot y^{2+4}$
$= 10\,x^4y^6$

Answers to Practice Problems
a. $20x^8$; b. x^{20}; c. $125x^3$; d. $16x^{11}$; e. $10\ x^4\ y^6$

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9.2 Adding and Subtracting Polynomials

1. Vocabulary:

- A variable is a quantity represented by a letter.
- A polynomial is the sum of terms that contain variables raised to positive integer or zero powers and that have no variables in any denominator.
- A term is one of the addends in an addition expression. For example, in the expression 2x + 4, the terms are 2x and 4.
- The parts of each term that are multiplied are the factors of the term. For example, in the term 2x from the example above, the factors are 2 and x.
- Like terms have the same variable factors raised to the same powers. For example, in the expression $2x^2+3x+7+3x^2+4x+9$, the $2x^2$ and $3x^2$ are like terms, the 3x and 4x are like terms, and the 7 and 9 are like terms.

2. Adding Polynomials: To add two polynomials, use the commutative and associative properties of addition to rewrite the sum so that like terms are grouped, and then use the distributive property to combine like terms.

Example 1: Simplify.

a. $(2x+7)+(4x-9)=(2x+7)+(4x+(-9))$

$=2x+7+4x+(-9)$

$=2x+4x+7+(-9)$

$=(2+4)x+(-2)$

$=6x-2$

b. $(5x-7)+(3x+9) = 5x+(-7)+3x+9$

$= (5x+3x)+[(-7)+9]$

$= 8x+2$

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c. $\left(2x^2+7x+4\right)+\left(4x^2+9x+8\right)$

$= 2x^2+7x+4+4x^2+9x+8$

$=(2x^2+4x^2)+(7x+9x)+(4+8)$

$= 6x^2+16x+12$

3. Negating Polynomials: If there is a negative sign directly preceding the parenthesis surrounding a polynomial, the negative sign applies to each term inside the parenthesis. Use the distributive property to distribute the negation to each term inside the parenthesis. You may think of the negative preceding the parenthesis as a –1, and use the rules for multiplying signed numbers.

Example 2: Simplify.

a. $-(5x+7)=-1(5x+7)$

$=(-1)(5x)+(-1)(7)$

$=-5x+(-7)$

$=-5x-7$

b. $-(3x+6) = (-1)(3x)+(-1)(6)$

$= -3x+(-6)$

$= -3x-6$

c. $-(3x-7) = (-1)(3x)+(-1)(-7)$

$= -3x+7$

d. $-(5x^2+7x-6) = (-1)(5x^2)+(-1)(7x)+(-1)(-6)$

$= -5x^2+(-7x)+6$

$= -5x^2-7x+6$

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4. Subtracting Polynomials: To subtract two polynomials, change the subtraction to addition of the opposite and then add.

Example: Simplify.

a. $(2x+3)-(5x+7)=(2x+3)+(-1)(5x+7)$
$=2x+3+(-1)(5x)+(-1)(7)$
$=2x+3+(-5x)+(-7)$
$=2x+(-5x)+3+(-7)$
$=(-3x)+(-4)$
$=-3x-4$

b. $(3x+5)-(7x+2)=3x+5+(-1)(7x+2)$
$=3x+5+(-1)(7x)+(-1)(2)$
$=3x+5+(-7x)+(-2)$
$=3x+(-7x)+5+(-2)$
$=-4x+3$

c. $(7x+8)-(6x-9)=(7x+8)+(-1)(6x-9)$
$=(7x+8)+(-1)(6x+(-9))$
$=7x+8+(-1)(6x)+(-1)(-9)$
$=7x+8+(-6x)+(9)$
$=x+17$

e. $(3x^2+5x-5)-(7x^2-2x-4)$
$=3x^2+5x-5+(-1)(7x^2)+(-1)(-2x)+(-1)(-4)$
$=3x^2+5x+(-5)+(-7x^2)+2x+4$
$=-4x^2+7x+(-1)$
$=-4x^2+7x-1$

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54. Evaluating Polynomials: To find the value of a polynomial at a given value of the variable, substitute the value of the variable into the polynomial everywhere the variable appears.

Example: Evaluate the given polynomial at the given value of the variable.

a. $2x-7$ at $x=-2$

$$2x-7=2(-2)-7$$
$$=-4-7$$
$$=-11$$

b. $2x^2+7x-5$ at $x=-3$

$$2x^2+7x-5 = 2(-3)^2+7(-3)-5$$
$$=2(9)+(-21)+(-5)$$
$$=18+(-21)+(-5)$$
$$=-3+(-5)$$
$$=-8$$

scratch work

$$(-3)^2=(-3)(-3)$$
$$=9$$

Practice Problems. Simplify each of the following:

a. $(2x-7)+(4x-9) = 2x-7+4x-9$
$$=(2x+4x)+[(-7)+(-9)]$$
$$=6x+(-16)$$
$$=6x-16$$

b. $(2x^2+7x+4)+(4x^2+9x+8)$
$$=2x^2+7x+4+4x^2+9x+8$$
$$=(2x^2+4x^2)+(7x+9x)+(4+8)$$
$$=6x^2+16x+12$$

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c. $-(6x+9) = -1(6x+9)$
$= (-1)(6x) + (-1)(9)$
$= -6x + (-9)$
$= -6x - 9$

d. $-(5x^2+7x-6) = (-1)(5x^2) + (-1)(7x) + (-1)(-6)$
$= -5x^2 + (-7x) + 6$
$= -5x^2 - 7x + 6$

d. $(6x-5)-(5x+2) = 6x + (-5) + (-1)(5x) + (-1)(2)$
$= 6x + (-5) + (-5x) + (-2)$
$= x + (-7)$
$= x - 7$

e. $(4x-6)-(7x-4) = 4x + (-6) + (-1)(7x) + (-1)(-4)$
$= 4x + (-6) + (-7x) + 4$
$= -3x + (-2)$
$= -3x - 2$

Evaluate the given polynomial at the given value of the variable.

f. $2x^2-5x-8$ at $x=-2$

$2x^2 - 5x - 8 = 2(-2)^2 - 5(-2) - 8$
$= 2(4) + 10 - 8$
$= 8 + 10 - 8$
$= 18 - 8$
$= 10$

Answers to Practice Problems
a. $6x-16$; b. $6x^2+16x+12$; c. $-6x-9$; d. $-5x^2-7x+6$
e. $-3x-2$; f. The value of the polynomial is 10.

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Section 6.1 Ratios

1. Definition of a Ratio: The ratio of two numbers is a fraction, where the first number in the ratio is the numerator and the second number in the ratio is the denominator. In symbols:

The ratio of a to b is $\frac{a}{b}$ where $b \neq 0$.

Example 1: Express the ratio of 16 to 48 as a fraction in lowest terms.

$$16:48 = \frac{16}{48} = \frac{2\cdot2\cdot2\cdot2}{2\cdot2\cdot2\cdot2\cdot3} = \frac{1}{3}$$

SDWK

16 = 2·2·2·2
48 = 2·2·2·2·3

Example 2: Express the ratio of $\frac{2}{3}$ to $\frac{4}{9}$ as a fraction in lowest terms.

$$\frac{2}{3}:\frac{4}{9} = \frac{\left(\frac{2}{3}\right)}{\left(\frac{4}{9}\right)} = \frac{2}{3}\cdot\frac{9}{4} = \frac{\cancel{2}\cdot\cancel{3}\cdot3}{\cancel{3}\cdot\cancel{2}\cdot2} = \frac{3}{2}$$

9 = 3·3
4 = 2·2

Example 3: Express the ratio of 0.08 to 0.12 as a fraction in lowest terms.

$$0.8 : 0.12 = \frac{0.08}{0.12} = \frac{\frac{8}{100}}{\frac{12}{100}} = \frac{8}{\cancel{100}}\cdot\frac{\cancel{100}}{12} = \frac{8}{12} = \frac{\cancel{2}\cdot\cancel{2}\cdot2}{\cancel{2}\cdot\cancel{2}\cdot3} = \frac{2}{3}$$

8 = 2·2·2
12 = 2·2·3

2. Applied Problems: In an applied problem, you may be asked to find the ratio of one quantity to a second quantity. The first quantity becomes the numerator of the fraction, and the second quantity becomes the denominator. Reduce the fraction to lowest terms.

Example 4: One cup of breakfast cereal contains 21 grams of carbohydrates and 2 grams of protein. Find the ratio of carbohydrates to protein.

$$\frac{\text{Carbohydrates}}{\text{protein}} = \frac{\text{21 grams of carbohydrates}}{\text{2 grams of protein}} = \frac{21}{2} = \frac{21 \div 2}{2 \div 2} = \frac{10.5}{1}$$

SDWK

$$\begin{array}{r} 10.5 \\ 2\,\overline{)\,21.0} \\ -2 \\ \hline 1 \\ -0 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

Practice Problems

a. Express the ratio of 18 to 27 as a fraction in lowest terms.

$18:27$

$= \frac{18}{27}$

$= \frac{2\cdot\cancel{3}\cdot\cancel{3}}{3\cdot\cancel{3}\cdot\cancel{3}}$

$= \frac{2}{3}$

SDWK

18: 2, 9; 9: 3, 3

27: 3, 9; 9: 3, 3

b. Express the ratio of $\frac{6}{7}$ to $\frac{12}{21}$ as a fraction in lowest terms.

$\frac{6}{7} : \frac{12}{21}$

$= \frac{\frac{6}{7}}{\frac{12}{21}}$

$= \frac{6}{7}\cdot\frac{21}{12}$

$= \frac{\cancel{2}\cdot3\cdot\cancel{3}\cdot\cancel{7}}{\cancel{7}\cdot\cancel{2}\cdot2\cdot\cancel{3}}$

$= \frac{3}{2}$

SDWK

6: 2, 3

12: 2, 6; 6: 2, 3

21: 3, 7

c. Express the ratio of 0.15 to 0.25 as a fraction in lowest terms

$0.15 : 0.25$

$= \frac{0.15}{0.25}$

$= \frac{\frac{15}{100}}{\frac{25}{100}}$

$= \frac{15}{100}\cdot\frac{100}{25}$

$= \frac{15}{25}$

$= \frac{3\cdot\cancel{5}}{5\cdot\cancel{5}}$

$= \frac{3}{5}$

SDWK

15: 3, 5

25: 5, 5

d. One cup of breakfast cereal contains 32 grams of carbohydrates and 3 grams of protein. Find the ratio of carbohydrates to protein.

32 grams of carbohydrates : 3 grams of protein

$= \frac{\text{32 grams of carbohydrates}}{\text{3 grams of protein}}$

$= \frac{32}{3}$

Answers to Practice Problems

a. $\frac{2}{3}$; *b.* $\frac{3}{2}$; *c.* $\frac{3}{5}$; *d.* $\frac{32}{3}$

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Section 6.3 Proportions

1. Definition of a Proportion: A statement that two ratios are equal is called a proportion. If $\frac{a}{b}$ and $\frac{c}{d}$ are two equal ratios, then the statement

$$\frac{a}{b} = \frac{c}{d}$$

is called a proportion. The "a" is the first term, "b" is the second term, "c" is the third term, and "d" is the fourth term. The **extremes** are the first and fourth terms and the **means** are the second and third terms.

Example 1: Name the means and the extremes in the given proportion.

$$\frac{5}{10} = \frac{6}{12}$$

Extremes: 5 & 12 | Means: 10 & 6

Extremes = Means
$5 \cdot 12 = 10 \cdot 6$
$60 = 60$
TRUE!

2. Fundamental Property of Proportions: If $\frac{a}{b} = \frac{c}{d}$, then ad = bc.

So, in any proportion the product of the means equals the product of the extremes.

Example 2: Verify that the given statements are proportions:

a. $\frac{8}{12} = \frac{10}{15}$

Extremes = Means
$8 \cdot 15 = 10 \cdot 12$

$120 = 120$
TRUE!

b. $\dfrac{\left(\frac{3}{5}\right)}{\left(\frac{7}{10}\right)} = \dfrac{\left(\frac{20}{7}\right)}{\left(\frac{10}{3}\right)}$

Extremes = Means
$\frac{3}{5} \cdot \frac{10}{3} = \frac{7}{10} \cdot \frac{20}{7}$

$\frac{\cancel{3} \cdot 2 \cdot \cancel{5}}{\cancel{5} \cdot \cancel{3}} = \frac{\cancel{7} \cdot 2 \cdot \cancel{2} \cdot \cancel{5}}{\cancel{2} \cdot \cancel{5} \cdot \cancel{7}}$

$\frac{2}{1} = \frac{2}{1}$

$2 = 2$

TRUE!

3DWK

$15 \times 8 = 120$

$10 = 2 \cdot 5$
$20 = 2 \cdot 2 \cdot 5$

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Example 3: Solve the proportion by finding the missing term.

a. $\frac{2}{3}=\frac{4}{x}$

Extremes Means

$2\cdot x = 3\cdot 4$

$2x = 12$

$\frac{1}{2}\cdot\frac{2x}{1}=\frac{1}{2}\cdot\frac{12}{1}$

$x=\frac{\cancel{2}\cdot 2\cdot 3}{\cancel{2}\cdot 1}$

$x=6$

check

$\frac{2}{3}=\frac{4}{(6)}$

$2\cdot 6 = 3\cdot 4$

$12=12$

TRUE!

SDWK

12 = 2 · 6, 6 = 2 · 3

$12 = 2\cdot 2\cdot 3$

ANS: The solution is 6.

b. $\frac{0.4}{1.2}=\frac{1}{x}$

Extremes Means

$0.4x = (1.2)\cdot 1$

$0.4x = 1.2$

$\frac{0.4x}{0.4}=\frac{1.2}{0.4}$

$x=3$

check

$\frac{0.4}{1.2}=\frac{1}{(3)}$

$0.4\cdot(3)=1.2\cdot(1)$

$1.2=1.2$

TRUE!

SDWK

$0.4\overline{)1.2}$

$4\overline{)12}$ = 3, -12, 0

$0.4 \times 3 = 1.2$

ANS: The solution is 3.

c. $\frac{n}{10}=\frac{\left(\frac{3}{5}\right)}{\left(\frac{3}{8}\right)}$

Extremes means

$\frac{n}{1}\cdot\left(\frac{3}{8}\right)=\frac{10}{1}\cdot\left(\frac{3}{5}\right)$

$\frac{3n}{8}=\frac{2\cdot\cancel{5}\cdot 3}{1\cdot\cancel{5}}$

$\frac{3n}{8}=\frac{6}{1}$

$\frac{8}{3}\cdot\frac{3n}{8}=\frac{8}{3}\cdot\frac{6}{1}$

$n=\frac{2\cdot 2\cdot 2\cdot 2\cdot\cancel{3}}{\cancel{3}\cdot 1}$

$n=16$

check!

$\frac{(16)}{10}=\frac{\frac{3}{5}}{\frac{3}{8}}$

$\frac{\cancel{2}\cdot 2\cdot 2\cdot 2}{\cancel{2}\cdot 5}=\frac{\cancel{3}}{5}\cdot\frac{8}{\cancel{3}}$

$\frac{8}{5}=\frac{8}{5}$

TRUE!

SDWK

10 = 2 · 5; 8 = 2 · 4, 4 = 2 · 2

$10=2\cdot 5$

$8=2\cdot 2\cdot 2$

$16=2\cdot 2\cdot 2\cdot 2$

16 = 2 · 8, 8 = 2 · 4, 4 = 2 · 2

ANS: The solution is 16.

3. Applications of Proportions: To set-up a proportion, let the first ratio compare two quantities in the first "situation", then let the second ratio compare "like" quantities in the second situation.

Example 4: In the first of 4 games of the season, a football team scores 68 points. At this rate, how many points will the team score in all 11 games? Let x = number of points scored in all 11 games

$$\frac{4 \text{ games}}{68 \text{ points}} = \frac{11 \text{ games}}{x \text{ points}}$$

$$\frac{4}{68} = \frac{11}{x}$$

Extremes Means

$$4 \cdot x = 68 \cdot 11$$

$$\frac{4x}{4} = \frac{68 \cdot 11}{4}$$

$$x = \frac{\cancel{2} \cdot \cancel{2} \cdot 17 \cdot 11}{\cancel{2} \cdot \cancel{2}}$$

$$x = 17 \cdot 11$$

$$x = 187$$

check

$$\frac{4}{68} = \frac{11}{(187)}$$

$$\frac{\cancel{2} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{2} \cdot 17} = \frac{\cancel{11} \cdot 1}{\cancel{11} \cdot 17}$$

$$\frac{1}{17} = \frac{1}{17}$$

TRUE!

SDWK

68 = 2 · 34, 34 = 2 · 17

4 = 2 · 2

68 = 2·2·17
4 = 2·2

$$\begin{array}{r} 17 \\ \times 11 \\ \hline 17 \\ +170 \\ \hline 187 \end{array}$$

ANS: The team will score 187 points in all 11 games.

Practice Problems:

Solve the proportion by finding the missing term.

a. $\frac{5}{4} = \frac{10}{x}$

Extremes Means

$$5 \cdot x = 4 \cdot 10$$

$$\frac{5x}{5} = \frac{4 \cdot 10}{5}$$

$$x = \frac{2 \cdot 2 \cdot 2 \cdot \cancel{5}}{\cancel{5} \cdot 1}$$

$$x = 8$$

check

$$\frac{5}{4} = \frac{10}{(8)}$$

$$\frac{5}{4} = \frac{\cancel{2} \cdot 5}{\cancel{2} \cdot 2 \cdot 2}$$

$$\frac{5}{4} = \frac{5}{4}$$

TRUE!

SDWK

10 = 2 · 5; 8 = 2 · 4, 4 = 2 · 2

10 = 2·5
8 = 2·2·2

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b. $\frac{0.2}{0.6} = \frac{4}{x}$

Extremes Means

$0.2 \cdot x = 4 \cdot (0.6)$

$0.2x = 2.4$

$\frac{0.2x}{0.2} = \frac{2.4}{0.2}$

$x = 12$

check!

$\frac{0.2}{0.6} = \frac{4}{(12)}$

$(0.2)(12) = (4)(0.6)$

$2.4 = 2.4$

TRUE!

SDWK

$$\begin{array}{r} 0.2 \\ \times 12 \\ \hline 4 \\ +20 \\ \hline 2.4 \end{array}$$

$$\begin{array}{r} 0.6 \\ \times 4 \\ \hline 2.4 \end{array}$$ 1 place

$0.2 \overline{)\,2.4}$

$$\begin{array}{r} 12 \\ 2\overline{)24} \\ -2 \\ \hline 4 \\ -4 \\ \hline 0 \end{array}$$

ANS: The solution is 12.

Solve the following application problems by setting up a proportion and then solving it.

c. If 12 oz. of a cereal contains 4 grams of sugar, how many grams of sugar are contained in 42 oz. of cereal? let x = number of grams of sugar in 42 oz of cereal

$\frac{12 \text{ oz. cereal}}{4 \text{ grams of sugar}} = \frac{42 \text{ oz. cereal}}{x \text{ grams of sugar}}$

$\frac{12}{4} = \frac{42}{x}$

Extremes Means

$12 \cdot x = 4 \cdot 42$

$12x = 4 \cdot 42$

$\frac{12x}{12} = \frac{4 \cdot 42}{12}$

$x = \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3} \cdot 7}{\cancel{2} \cdot \cancel{2} \cdot \cancel{3}}$

$x = 14$

check!

$\frac{12}{4} = \frac{42}{14}$

$12 \cdot 14 = 4 \cdot 42$

$168 = 168$

TRUE!

SDWK

$4 = 2 \cdot 2$

$42 = 2 \cdot 3 \cdot 7$

$12 = 2 \cdot 2 \cdot 3$

42 → 7, 6 → 2, 3

12 → 2, 6 → 2, 3

$$\begin{array}{r} 12 \\ \times 14 \\ \hline 48 \\ +120 \\ \hline 168 \end{array}$$

$$\begin{array}{r} 42 \\ \times 4 \\ \hline 168 \end{array}$$

d. An 8 oz. serving of a certain cereal contains 256 calories. How many calories are contained in a 20 oz. serving?

Let x = number of calories in 20 oz. serving of cereal

$\frac{8 \text{ oz. cereal}}{256 \text{ calories}} = \frac{20 \text{ oz. cereal}}{x \text{ calories}}$

$\frac{8}{256} = \frac{20}{x}$

Extremes Means

$8 \cdot x = 256 \cdot 20$

$\frac{8x}{8} = \frac{256 \cdot 20}{8}$

$x = \frac{\cancel{2} \cdot 128 \cdot \cancel{2} \cdot \cancel{2} \cdot 5}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2}}$

$x = 128 \cdot 5$

$x = 640$

check!

$\frac{8}{256} = \frac{20}{640}$

$8 \cdot 640 = 20 \cdot 256$

$5{,}120 = 5{,}120$

TRUE!

SDWK

$8 = 2 \cdot 2 \cdot 2$

$20 = 2 \cdot 2 \cdot 5$

$256 = 2 \cdot 128$

8 → 2, 4 → 2, 2

20 → 2, 10 → 2, 5

256 → 2, 128

$$\begin{array}{r} 640 \\ \times 8 \\ \hline 5120 \end{array}$$

$$\begin{array}{r} 256 \\ \times 20 \\ \hline 000 \\ +5120 \\ \hline 5{,}120 \end{array}$$

Answers to Practice Problems:

a. {8}; b. {12}; c. 42 oz. of cereal contains 14 g. of sugar; d. A 20 oz. serving of cereal contains 640 calories.

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Section 7.2 Basic Percent Problems

1. Vocabulary: The following translations of certain English words to mathematical symbols will be helpful to you in solving basic percent problems.

English word	Math symbol
is	=
of	•
a number	n
what number	n
what percent	n OR $\boxed{\frac{n}{100}}$ ★★

2. Solving Percent Problems Using Equations: To solve percent problems using equations, translate the sentences into equations and then solve the equations. Percent problems translate into three types of word problems.

Type One: What number is 10% of 80?

Example 1: Translate the given percent problem into an equation, and then solve the equation.

What number is 10% of 80?

let n = unknown number

$$n = (0.10)\cdot(80)$$
$$n = 8$$

ANS: 8 is 10% of 80.

check:
$(8) = (0.10)\cdot(80)$
$8 = 8$
TRUE!

10% = 0.10

$$\begin{array}{r} 80 \\ \times\ 0.10 \\ \hline 00 \\ +800 \\ \hline 8.00 \end{array}$$

2 places

Type Two: What percent of 80 is 20?

Example 2: Translate the given percent problem into an equation, and then solve the equation.

Fourteen is what percent of 70?

$$14 = \left(\frac{n}{100}\right)\cdot\left(\frac{70}{1}\right)$$
$$14 = \frac{70n}{100}$$
$$14 = \frac{7}{10}n$$
$$\frac{10}{7}\cdot\frac{14}{1} = \frac{10}{7}\cdot\frac{7}{10}n$$
$$\frac{2\cdot5\cdot2\cdot\cancel{7}}{\cancel{7}\cdot1} = n$$
$$2\cdot2\cdot5 = n$$
$$20 = n$$

ANS: 14 is 20% of 70.

check:
$14 = \frac{(20)}{100}\cdot\frac{70}{1}$
$14 = \frac{2\cdot2\cdot5\cdot7\cdot2\cdot5}{2\cdot5\cdot2\cdot5}$
$14 = 2\cdot7$
$14 = 14$
TRUE!

SPWK
$10 = 2\cdot5$
$14 = 2\cdot7$
$70 = 7\cdot2\cdot5$
$100 = 2\cdot5\cdot2\cdot5$

70 → 7, 10 → 2, 5
100 → 10, 10 → 2, 5, 2, 5

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

Type Three: Fifty is 20% of what number?

Example 3: Translate the given percent problem into an equation, and then solve the equation.

Fifty is 20% of what number?

Let n = unknown number

$50 = (0.2)\cdot n$

$50 = \frac{2}{10}\cdot n$

$\frac{10}{2}\cdot\frac{50}{1} = \frac{10}{2}\cdot\frac{2}{10}\cdot n$

$\frac{\cancel{2}\cdot 5\cdot 50}{\cancel{2}\cdot 1} = n$

$250 = n$

check:

$50 = (0.2)(250)$

$50 = 50$

TRUE!

sdwk

$20\% = 0.20$

$= 0.2$

10 → 2 5

250
× 0.2 1 place
50.0

ANS: 50 is 20% of 250.

3. Vocabulary: Percent problems involve three quantities: a percent, a base (usually a beginning quantity or a quantity used as a base for comparison), and a final amount (usually the final quantity or the ending quantity). In general, the percent times the base equals the amount. In symbols: $A = P \bullet B$.

Example 4: In each problem, identify the percent, the base, and the final amount.

a. 50% of 40 is 20 — The percent is 50%, the base is 40 and the amount is 20.

$P = 50\% = 0.5$
$B = 40$
$A = 20$

b. 10% of 80 is 8 — The percent is 10%, the base is 80, and the amount is 8.

$P = 10\% = 0.1$
$B = 80$
$A = 8$

c. 15 is 50% of 30 — The percent is 50%, the base is 30, and the amount is 15.

$P = 50\% = 0.5$
$B = 30$
$A = 15$

4. Solving Percent Problems Using Proportions: Write the percent as one of the fractions in the proportion and $\frac{\text{amount}}{\text{base}}$ as the other fraction. Equate the two fractions, and solve using the Fundamental Property of Proportions.

use $A = P\cdot B$

OR

$\boxed{\frac{A}{B} = P}$

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

Example: Solve each of the following percent problems using proportions.

Let n = unknown number

a. What number is 15% of 63? , use $A = P \cdot B$ or $\boxed{\frac{A}{B} = P}$

$\boxed{\begin{array}{l} P = 15\% = 0.15 \\ B = 63 \\ A = n \end{array}}$

$\frac{n}{63} = 0.15$

$\frac{63}{1} \cdot \frac{n}{63} = \frac{63}{1} \cdot (0.15)$

$n = 9.45$

check:

$\frac{(9.45)}{63} = 0.15$

$0.15 = 0.15$

TRUE!

SDWK

63×0.15 (2 places) $= 315 + 630 = 9.45$

$9.45 \div 63 = 0.15$; -63, 315, -315, 0

ANS: 9.45 is 15% of 63.

Let $\frac{n}{100}$ = unknown percent

b. What percent of 42 is 21? , use $A = P \cdot B$, or $\boxed{\frac{A}{B} = P}$

$\boxed{\begin{array}{l} P = \frac{n}{100} \\ B = 42 \\ A = 21 \end{array}}$

$\frac{21}{42} = \frac{n}{100}$

$\frac{3 \cdot 7}{2 \cdot 3 \cdot 7} = \frac{n}{100}$

$\frac{1}{2} = \frac{n}{100}$

$\frac{100}{1} \cdot \frac{1}{2} = \frac{100}{1} \cdot \frac{n}{100}$

$50 = n$

check:

$\frac{21}{42} = \frac{(50)}{100}$

$21 \cdot 100 = 50 \cdot 42$

$2{,}100 = 2{,}100$

TRUE!

SDWK

$21 = 3 \cdot 7$

$42 = 2 \cdot 21 = 2 \cdot 3 \cdot 7$

50×42: $100 + 2000 = 2{,}100$

ANS: 50% of 42 is 21.

c. 25 is 40% of what number? Let n = unknown number use $A = P \cdot B$, or $\boxed{\frac{A}{B} = P}$

$\boxed{\begin{array}{l} P = 40\% = 0.4 \\ B = n \\ A = 25 \end{array}}$

$\frac{25}{n} = 0.4$

$\frac{n}{1} \cdot \frac{25}{n} = \frac{n}{1} \cdot (0.4)$

$25 = 0.4n$

$\frac{25}{0.4} = \frac{0.4n}{0.4}$

$62.5 = n$

check:

$\frac{25}{(62.5)} = 0.4$

$\frac{25}{62.5} = \frac{4}{10}$

$25 \cdot 10 = 4 \cdot (62.5)$

$250 = 250$

TRUE!

$0.4\overline{)25.0}$

$4\overline{)250.0} = 62.5$; -24, 10, -8, 20, -20, 0

ANS: 25 is 40% of 62.5.

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

Practice Problems. Solve each of the following percent problems using either of the techniques given above.

a. Fifty percent of 120 is what number?

Let n = unknown number

use $A = P \cdot B$

$P = 50\% = 0.5$
$B = 120$
$A = n$

$n = (0.5) \cdot 120$
$n = 60$

check:
$(60) = (0.5) \cdot 120$
$60 = 60$
TRUE!

SDWK
$120 \times 0.5 = 60.0$

ANS: 50% of 120 is 60.

b. Three percent of what number is 6?

Let n = unknown number

use $A = P \cdot B$

$P = 3\% = 0.03$
$B = n$
$A = 6$

$6 = (0.03) \cdot n$
$\frac{6}{0.03} = \frac{0.03n}{0.03}$
$200 = n$

check:
$6 = (0.03) \cdot (200)$
$6 = 6$
TRUE!

SDWK
$0.03 \overline{)6.00}$
$3 \overline{)600}$ = 200

ANS: 3% of 200 is 6.

c. 34 is what percent of 85?

Let $\frac{n}{100}$ = unknown Percent

use $A = P \cdot B$, or $\boxed{\frac{A}{B} = P}$

$P = \frac{n}{100}$
$B = 85$
$A = 34$

$\frac{34}{85} = \frac{n}{100}$

$\frac{2 \cdot \cancel{17}}{5 \cdot \cancel{17}} = \frac{n}{100}$

$\frac{2}{5} = \frac{n}{100}$

$\frac{100}{1} \cdot \frac{2}{5} = \frac{100}{1} \cdot \frac{n}{100}$

$\frac{5 \cdot 20 \cdot 2}{\cancel{5} \cdot 1} = n$
$40 = n$

check:
$\frac{34}{85} = \frac{(40)}{100}$
$34 \cdot 100 = 40 \cdot 85$
$3{,}400 = 3{,}400$
TRUE!

SDWK
34 → 2 17; 85 → 5 17; 100 → 5 20

40×85: 200 + 3200 = 3,400

ANS: 34 is 40% of 85.

Answers to Practice Problems:
a. 60; b. 200; c. 40%

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

Section 7.3 General Applications of Percents

1. Vocabulary: The following translations of certain English words to mathematical symbols will be helpful to you in solving word problems that involve percents.

English word	Math symbol
is	=
of	•
a number	n
what number	n
what percent	n → $\boxed{\frac{n}{100}}$ unknown percent

Percent problems involve three quantities: a percent, a base (usually a beginning quantity or a quantity used as a base for comparison), and a final amount (usually the final quantity or the ending quantity). In general, the percent times the base equals the amount. In symbols: $A = P \bullet B$.

2. Applications: To solve percent problems, translate the word problems into equations and then solve the equations.

Example 1: Solve each application problem by setting up an equation, and then solving the equation. Use the proper format: Write a statement identifying the quantity that your variable stands for, set up an equation, solve the equation, and then write your solution in English words.

a. Forty percent of the students in a particular class made an A or B on Test One. If there were 35 students in the class, how many students made an A or B?

Statement: 40% of 35 students made an A or B.
Let n = number of students who made an A or B on Test One.
Use $A = P \cdot B$, with $P = 40\% = 0.4$, $B = 35$, and $A = n$

So, $n = (0.4)\cdot(35)$
$n = 14$

Check!
$(14) = (0.4)(35)$ or $\frac{14}{35} = 0.4$
$14 = 14$ $0.4 = 0.4$
TRUE! TRUE!

Spwk
$35 \times 0.4 = 14.0$ (1 place)
$14.00 \div 35 = 0.4$; -140; 0

ANS: Out of 35 students in the class, 14 students made an A or B on Test One.

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

20/20

b. A certain meal contains 850 calories, 255 of which come from protein. What percentage of the calories is from protein?

statement: 255 calories from protein is what percent of the 850 total calories?

Let $\frac{n}{100}$ = unknown percentage

use $A = P \cdot B$, with $P = \frac{n}{100}$, $B = 850$, and $A = 255$.

So, $(255) = \left(\frac{n}{100}\right) \cdot \left(\frac{850}{1}\right)$

$255 = \frac{850n}{100}$

$255 = 8.5n$

$\frac{255}{8.5} = \frac{8.5n}{8.5}$

$30 = n$

check

$n = 30$

$255 = \frac{(30)}{100} \cdot \frac{850}{1}$

$255 = (0.3) \cdot 850$

$255 = 255$

TRUE!

SD wk

$8.5\overline{)255.0}$

$$\begin{array}{r} 30. \\ 85\overline{)2550.} \\ -255 \\ \hline 0 \\ -0 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 85 \\ \times 3 \\ \hline 255 \end{array}$$

$$\begin{array}{r} 850 \\ \times 0.3 \\ \hline 255.0 \end{array}$$ 1 place

ANS: Out of the 850 total calories, 30% come from protein.

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague